Answer the following questions

1- Suppose there are 5 black, 10 white, and 15 red marbles in an urn. You reach in and randomly select six marbles without replacement. What is the probability that you pick exactly two red marbles?   Find E(X) and var(X)

n = 6, N = 30, k = 15, therefore $P(X = 2) = [\,^{15}C_2\,]\,[\,^{15}C_4\,]\,/\,[\,^{30}C_6\,]$

$E(X) = n\,(k/N) = 6(15/30) = 3$, and $V(X) = (\dfrac{N-n}{N-1})n(\dfrac{k}{N})(1-\dfrac{k}{N}) = \dfrac{36}{29}$

2- Two fair dice are rolled, what is the probability that the first turns up six, given that the sum is k, for each k from two through 12?

A={ first turns up six }= { (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

B = { the sum is k, for each k from two through 12}, therefore P(A/B) = 0 for k =2,3,…,6.

At k = 7, B = { (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)}, thus P(A/B) = 1/6

k = 8, B = { (2,6), (3,5), (4,4), (5,3), (6,2) }, thus P(A/B) = 1/5

k = 9, B = { (3,6), (4,5), (5,4), (6,3) }, thus P(A/B) = ¼

k = 10, B = { (4,6), (5,5), (6,4) }, thus P(A/B) = 1/3

k = 11, B = { (5,6), (6,5) }, thus P(A/B) = ½

k = 12, B = {(6,6)}, thus P(A/B) = 1

3- The density function of X is given by $f(x) = \begin{cases} a - bx^2 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$,

If $E(X) = 3/5$, find a and b.

Since $\int_0^1 x(a - bx^2)dx = 0.6$, therefore $[a\dfrac{x^2}{2} - b\dfrac{x^4}{4}]_0^1 = 0.6$, thus $2a - b = 2.4$, but

$\int_0^1 (a - bx^2)dx = 1$, therefore $[ax - b\dfrac{x^3}{3}]_0^1 = 1$, thus $3a - b = 3$, so $a = 0.6$ and $b = -1.2$

Good Luck

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